SOLUTIONS TO CONCEPTS CHAPTER - 20

1. Given that,

Refractive index of flint glass = μ_f = 1.620

Refractive index of crown glass = μ_c = 1.518

Refracting angle of flint prism = $A_f = 6.0^{\circ}$

For zero net deviation of mean ray

$$(\mu_f - 1)A_f = (\mu_c - 1) A_c$$

$$\Rightarrow A_c = \frac{\mu_f - 1}{\mu_c - 1} A_f = \frac{1.620 - 1}{1.518 - 1} (6.0)^\circ = 7.2^\circ$$

$$\mu_r$$
 = 1.56, μ_y = 1.60, and μ_v = 1.68

(a) Dispersive power =
$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.68 - 1.56}{1.60 - 1} = 0.2$$

(b) Angular dispersion = $(\mu_v - \mu_r)A = 0.12 \times 6^\circ = 7.2^\circ$

The focal length of a lens is given by

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow (\mu - 1) = \frac{1}{f} \times \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2} \right)} = \frac{K}{f} \qquad ...(1)$$
So, $\mu_r - 1 = \frac{K}{100} \qquad ...(2)$

$$\mu_y - 1 = \frac{K}{98} \qquad ...(3)$$

So,
$$\mu_r - 1 = \frac{K}{100}$$

$$\mu_y - 1 = \frac{K}{98}$$

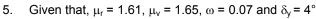
And
$$\mu_v - 1 = \frac{K}{96}$$

So, Dispersive power =
$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \frac{\frac{K}{96} - \frac{K}{100}}{\frac{K}{98}} = \frac{98 \times 4}{9600} = 0.0408$$

4. Given that,
$$\mu_v - \mu_r = 0.014$$

Again,
$$\mu_y = \frac{\text{Re al depth}}{\text{Apparent depth}} = \frac{2.00}{1.30} = 1.515$$

So, dispersive power =
$$\frac{\mu_{v} - \mu_{r}}{\mu_{v} - 1} = \frac{0.014}{1.515 - 1} = 0.027$$



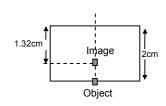
Now,
$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

$$\Rightarrow$$
 0.07 = $\frac{1.65 - 1.61}{\mu_v - 1}$

$$\Rightarrow \mu_y - 1 = \frac{0.04}{0.07} = \frac{4}{7}$$

Again,
$$\delta = (\mu - 1) A$$

$$\Rightarrow A = \frac{\delta_y}{\mu_y - 1} = \frac{4}{(4/7)} = 7^{\circ}$$



Given that, $\delta_r = 38.4^\circ$, $\delta_v = 38.7^\circ$ and $\delta_v = 39.2^\circ$

Dispersive power =
$$\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \frac{\left(\frac{\delta_v}{A}\right) - \left(\frac{\delta_r}{A}\right)}{\left(\frac{\delta_v}{A}\right)} \qquad [\because \delta = (\mu - 1) A]$$

$$= \frac{\delta_{v} - \delta_{r}}{\delta_{v}} = \frac{39.2 - 38.4}{38.7} = 0.0204$$

Two prisms of identical geometrical shape are combined.

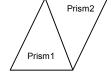
Let A = Angle of the prisms

$$\mu'_{v}$$
 = 1.52 and μ_{v} = 1.62, δ_{v} = 1°

$$\delta_{v} = (\mu_{v} - 1)A - (\mu'_{v} - 1)A$$
 [since A = A']

$$\Rightarrow \delta_v = (\mu_v - \mu'_v)A$$

$$\Rightarrow$$
 A = $\frac{\delta_{v}}{\mu_{v} - \mu'_{v}} = \frac{1}{1.62 - 1.52} = 10^{\circ}$



Total deviation for yellow ray produced by the prism combination is

$$\delta_y = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2 \; \delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{cy} - 1)A'$$

Similarly the angular dispersion produced by the combination is

$$\delta_{v} - \delta_{r} = [(\mu_{vc} - 1)A - (\mu_{vf} - 1)A' + (\mu_{vc} - 1)A] - [(\mu_{rc} - 1)A - (\mu_{rf} - 1)A' + (\mu_{r} - 1)A)]$$

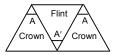
$$= 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$$

(a) For net angular dispersion to be zero,

$$\delta_{\rm v} - \delta_{\rm r} = 0$$

$$\Rightarrow$$
 2(μ_{vc} – 1)A = (μ_{vf} – 1)A'

$$\Rightarrow \frac{\mathsf{A}'}{\mathsf{A}} = \frac{2(\mu_{\mathsf{cv}} - \mu_{\mathsf{rc}})}{(\mu_{\mathsf{vf}} - \mu_{\mathsf{rf}})} = \frac{2(\mu_{\mathsf{v}} - \mu_{\mathsf{r}})}{(\mu'_{\mathsf{v}} - \mu'_{\mathsf{r}})}$$



(b) For net deviation in the yellow ray to be zero

$$\delta_y = 0$$

$$\stackrel{,}{\Rightarrow} 2(\mu_{cy}-1)A = (\mu_{fy}-1)A$$

$$\Rightarrow 2(\mu_{cy} - 1)A = (\mu_{fy} - 1)A'$$

$$\Rightarrow \frac{A'}{A} = \frac{2(\mu_{cy} - 1)}{(\mu_{fy} - 1)} = \frac{2(\mu_{y} - 1)}{(\mu'_{y} - 1)}$$

Given that, $\mu_{cr} = 1.515$, $\mu_{cv} = 1.525$ and $\mu_{fr} = 1.612$, $\mu_{fv} = 1.632$ and A = 5°

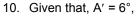
Since, they are similarly directed, the total deviation produced is given by,

$$\delta = \delta_c + \delta_r = (\mu_c - 1)A + (\mu_r - 1)A = (\mu_c + \mu_r - 2)A$$

So, angular dispersion of the combination is given by,

$$\delta_{v} - \delta_{y} = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr} - 2)A$$

=
$$(\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A$$
 = $(1.525 + 1.632 - 1.515 - 1.612)$ 5 = 0.15°



$$\omega' = 0.07$$
,

$$\mu'_{y} = 1.50$$

$$\omega = 0.08, \qquad \mu_{y} = 1.60$$



The combination produces no deviation in the mean ray.

(a)
$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' = 0$$

$$\Rightarrow$$
 (1.60 – 1)A = ((1.50 – 1)A'

$$\Rightarrow$$
 A = $\frac{0.50 \times 6^{\circ}}{0.60}$ = 5°



(b) When a beam of white light passes through it,

Net angular dispersion =
$$(\mu_v - 1)\omega A - (\mu'_v - 1)\omega' A'$$

$$\Rightarrow (1.60-1)(0.08)(5^{\circ}) - \ (1.50-\ 1)(0.07)(6^{\circ})$$

$$\Rightarrow$$
 0.24° - 0.21° = 0.03°

(c) If the prisms are similarly directed,

$$\delta_y = (\mu_y - 1)A + (\mu'_y - 1)A$$

$$= (1.60 - 1)5^{\circ} + (1.50 - 1)6^{\circ} = 3^{\circ} + 3^{\circ} = 6^{\circ}$$



(d) Similarly, if the prisms are similarly directed, the net angular dispersion is given by, $\delta_{v} - \delta_{r} = (\mu_{y} - 1)\omega A - (\mu'_{y} - 1)\omega' A' = 0.24^{\circ} + 0.21^{\circ} = 0.45^{\circ}$



11. Given that, $\mu'_v - \mu'_r = 0.014$ and $\mu_v - \mu_r = 0.024$ A' = 5.3° and A = 3.7°

- (a) When the prisms are oppositely directed, angular dispersion = $(\mu_v \mu_r)A (\mu'_v \mu'_r)A'$ = $0.024 \times 3.7^{\circ} 0.014 \times 5.3^{\circ} = 0.0146^{\circ}$
- (b) When they are similarly directed, angular dispersion = $(\mu_v \mu_r)A + (\mu'_v \mu'_r)A'$ = 0.024 × 3.7° + 0.014 × 5.3° = 0.163°





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