

$$\begin{aligned} 16. \quad K_{AB} &= 50 \text{ J/m-s-}^{\circ}\text{C} & \theta_A &= 40^{\circ}\text{C} \\ K_{BC} &= 200 \text{ J/m-s-}^{\circ}\text{C} & \theta_B &= 80^{\circ}\text{C} \\ K_{AC} &= 400 \text{ J/m-s-}^{\circ}\text{C} & \theta_C &= 80^{\circ}\text{C} \end{aligned}$$

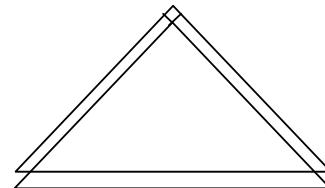
$$\text{Length} = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$$

$$(a) \frac{Q_{AB}}{t} = \frac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 1 \text{ W.}$$

$$(b) \frac{Q_{AC}}{t} = \frac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 800 \times 10^{-2} = 8$$

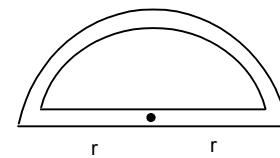
$$(c) \frac{Q_{BC}}{t} = \frac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}} = 0$$



$$17. \quad \text{We know } Q = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$Q_1 = \frac{KA(\theta_1 - \theta_2)}{d_1}, \quad Q_2 = \frac{KA(\theta_1 - \theta_2)}{d_2}$$

$$\frac{Q_1}{Q_2} = \frac{\frac{KA(\theta_1 - \theta_1)}{\pi r}}{\frac{KA(\theta_1 - \theta_1)}{2r}} = \frac{2r}{\pi r} = \frac{2}{\pi}$$



18. The rate of heat flow per sec.

$$= \frac{dQ_A}{dt} = KA \frac{d\theta}{dt}$$

The rate of heat flow per sec.

$$= \frac{dQ_B}{dt} = KA \frac{d\theta_B}{dt}$$

This part of heat is absorbed by the red.

$$\frac{Q}{t} = \frac{ms\Delta\theta}{dt} \quad \text{where } \frac{d\theta}{dt} = \text{Rate of net temp. variation}$$

$$\Rightarrow \frac{msd\theta}{dt} = KA \frac{d\theta_A}{dt} - KA \frac{d\theta_B}{dt} \quad \Rightarrow ms \frac{d\theta}{dt} = KA \left(\frac{d\theta_A}{dt} - \frac{d\theta_B}{dt} \right)$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 1 \times 10^{-4} (5 - 2.5) \text{ }^{\circ}\text{C/cm}$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 10^{-4} \times 2.5$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}} \text{ }^{\circ}\text{C/m} = 1250 \times 10^{-2} = 12.5 \text{ }^{\circ}\text{C/m}$$

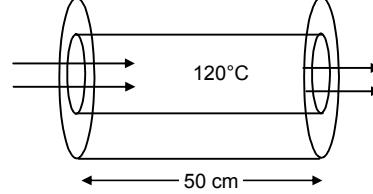
19. Given

$$K_{rubber} = 0.15 \text{ J/m-s-}^{\circ}\text{C} \quad T_2 - T_1 = 90^{\circ}\text{C}$$

We know for radial conduction in a Cylinder

$$Q = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$$

$$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)} = 232.5 \approx 233 \text{ J/s.}$$



$$20. \quad \frac{dQ}{dt} = \text{Rate of flow of heat}$$

Let us consider a strip at a distance r from the center of thickness dr .

$$\frac{dQ}{dt} = \frac{K \times 2\pi r d \times d\theta}{dr} \quad [d\theta = \text{Temperature diff across the thickness dr}]$$

$$\Rightarrow C = \frac{K \times 2\pi r d \times d\theta}{dr} \quad \left[C = \frac{d\theta}{dr} \right]$$

$$\Rightarrow C \frac{dr}{r} = K 2\pi d d\theta$$

Integrating

$$\Rightarrow C \int_{r_1}^{r_2} \frac{dr}{r} = K 2\pi d \int_{\theta_1}^{\theta_2} d\theta \quad \Rightarrow C [\ln r]_{r_1}^{r_2} = K 2\pi d (\theta_2 - \theta_1)$$

$$\Rightarrow C(\ln r_2 - \ln r_1) = K 2\pi d (\theta_2 - \theta_1) \Rightarrow C \ln \left(\frac{r_2}{r_1} \right) = K 2\pi d (\theta_2 - \theta_1)$$

$$\Rightarrow C = \frac{K 2\pi d (\theta_2 - \theta_1)}{\ln(r_2 / r_1)}$$

21. $T_1 > T_2$
 $A = \pi(R_2^2 - R_1^2)$

$$\text{So, } Q = \frac{KA(T_2 - T_1)}{l} = \frac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$$

Considering a concentric cylindrical shell of radius 'r' and thickness 'dr'. The radial heat flow through the shell

$$H = \frac{dQ}{dt} = -KA \frac{d\theta}{dt} \quad [(-)\text{ve because as } r \text{ increases } \theta \text{ decreases}]$$

$$A = 2\pi rl \quad H = -2\pi rl K \frac{d\theta}{dt}$$

$$\text{or } \int_{R_1}^{R_2} \frac{dr}{r} = -\frac{2\pi L K}{H} \int_{T_1}^{T_2} d\theta$$

Integrating and simplifying we get

$$H = \frac{dQ}{dt} = \frac{2\pi K L (T_2 - T_1)}{\ln(R_2 / R_1)} = \frac{2\pi K L (T_2 - T_1)}{\ln(R_2 / R_1)}$$

22. Here the thermal conductivities are in series,

$$\therefore \frac{\frac{K_1 A(\theta_1 - \theta_2)}{l_1} \times \frac{K_2 A(\theta_1 - \theta_2)}{l_2}}{\frac{K_1 A(\theta_1 - \theta_2)}{l_1} + \frac{K_2 A(\theta_1 - \theta_2)}{l_2}} = \frac{K A(\theta_1 - \theta_2)}{l_1 + l_2}$$

$$\Rightarrow \frac{\frac{K_1}{l_1} \times \frac{K_2}{l_2}}{\frac{K_1}{l_1} + \frac{K_2}{l_2}} = \frac{K}{l_1 + l_2}$$

$$\Rightarrow \frac{K_1 K_2}{K_1 l_2 + K_2 l_1} = \frac{K}{l_1 + l_2} \Rightarrow K = \frac{(K_1 K_2)(l_1 + l_2)}{K_1 l_2 + K_2 l_1}$$

23. $K_{Cu} = 390 \text{ W/m}^\circ\text{C}$ $K_{St} = 46 \text{ W/m}^\circ\text{C}$

Now, Since they are in series connection,

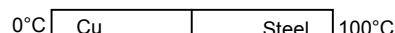
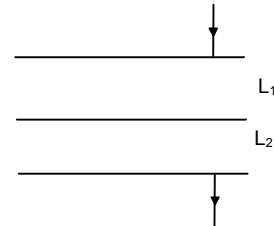
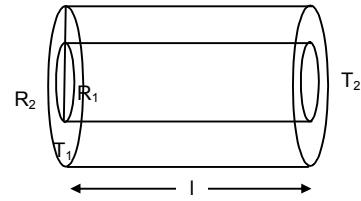
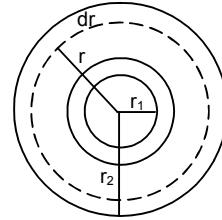
So, the heat passed through the crossections in the same.

So, $Q_1 = Q_2$

$$\text{Or } \frac{K_{Cu} \times A \times (\theta - 0)}{l} = \frac{K_{St} \times A \times (100 - \theta)}{l}$$

$$\Rightarrow 390(\theta - 0) = 46 \times 100 - 46 \theta \Rightarrow 436 \theta = 4600$$

$$\Rightarrow \theta = \frac{4600}{436} = 10.55 \approx 10.6^\circ\text{C}$$



32. The temp at the both ends of bar F is same

Rate of Heat flow to right = Rate of heat flow through left

$$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$$

$$\Rightarrow \frac{K_A(T_1 - T)A}{L} + \frac{K_C(T_1 - T)A}{L} = \frac{K_B(T - T_2)A}{L} + \frac{K_D(T - T_2)A}{L}$$

$$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$$

$$\Rightarrow T_1 - T = 2T - 2T_2$$

$$\Rightarrow T = \frac{T_1 + 2T_2}{3}$$

$$33. \tan \phi = \frac{r_2 - r_1}{L} = \frac{(y - r_1)}{x}$$

$$\Rightarrow xr_2 - xr_1 = yL - r_1L$$

Differentiating wr to 'x'

$$\Rightarrow r_2 - r_1 = \frac{Ldy}{dx} - 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{r_2 - r_1}{L} \Rightarrow dx = \frac{dyL}{(r_2 - r_1)} \quad \dots(1)$$

$$\text{Now } \frac{Q}{T} = \frac{K\pi y^2 d\theta}{dx} \Rightarrow \frac{\theta dx}{T} = k\pi y^2 d\theta$$

$$\Rightarrow \frac{\theta L dy}{r_2 r_1} = K\pi y^2 d\theta \quad \text{from}(1)$$

$$\Rightarrow d\theta \frac{QL dy}{(r_2 - r_1)K\pi y^2}$$

Integrating both side

$$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \frac{QL}{(r_2 - r_1)K\pi} \int_{r_1}^{r_2} \frac{dy}{y}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{-1}{y} \right]_{r_1}^{r_2}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{r_2 - r_1}{r_1 + r_2} \right]$$

$$\Rightarrow Q = \frac{K\pi r_1 r_2 (\theta_2 - \theta_1)}{L}$$

$$34. \frac{d\theta}{dt} = \frac{60}{10 \times 60} = 0.1^\circ\text{C/sec}$$

$$\frac{dQ}{dt} = \frac{KA}{d} (\theta_1 - \theta_2)$$

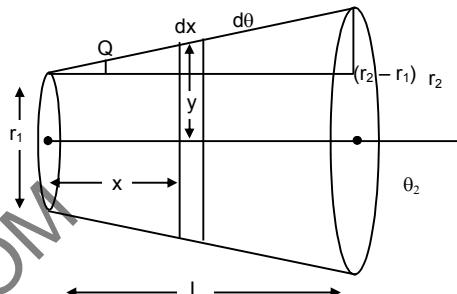
$$= \frac{KA \times 0.1}{d} + \frac{KA \times 0.2}{d} + \dots + \frac{KA \times 60}{d}$$

$$= \frac{KA}{d} (0.1 + 0.2 + \dots + 60) = \frac{KA}{d} \times \frac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$$

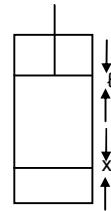
$$[\therefore a + 2a + \dots + na = n/2 \{2a + (n-1)a\}]$$

$$= \frac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \frac{200 \times 10^{-2} \times 300 \times 60.1}{20}$$

$$= 3 \times 10 \times 60.1 = 1803 \text{ w} \approx 1800 \text{ w}$$



$$\begin{aligned}
 38. \quad \frac{Q}{t} &= \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_p dT}{dt} = \frac{KA(T_s - T_0)}{x} \\
 \Rightarrow \frac{n(5/2)RdT}{dt} &= \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_s - T_0) \\
 \Rightarrow \frac{dT}{(T_s - T_0)} &= -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_s - T_0)_{T_0}^T = -\frac{2KAt}{5nRx} \\
 \Rightarrow \ln \frac{T_s - T}{T_s - T_0} &= -\frac{2KAt}{5nRx} \Rightarrow T_s - T = (T_s - T_0)e^{-\frac{2KAt}{5nRx}} \\
 \Rightarrow T &= T_s - (T_s - T_0)e^{-\frac{2KAt}{5nRx}} = T_s + (T_s + T_0)e^{+\frac{2KAt}{5nRx}} \\
 \Rightarrow \Delta T &= T - T_0 = (T_s - T_0) + (T_s - T_0)e^{+\frac{2KAt}{5nRx}} = (T_s - T_0) + \left(1 + e^{\frac{2KAt}{5nRx}}\right) \\
 \Rightarrow \frac{P_a AL}{nR} &= (T_s - T_0) + \left(1 + e^{\frac{2KAt}{5nRx}}\right) \quad [P_a dv = nR dt \quad P_a Al = nR dt \quad dT = \frac{P_a Al}{nR}] \\
 \Rightarrow L &= \frac{nR}{P_a A}(T_s - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)
 \end{aligned}$$



$$\begin{aligned}
 39. \quad A &= 1.6 \text{ m}^2, \quad T = 37^\circ\text{C} = 310 \text{ K}, \quad \sigma = 6.0 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \\
 &\text{Energy radiated per second} \\
 &= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887 \text{ J} \\
 40. \quad A &= 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2 \quad T = 20^\circ\text{C} = 293 \text{ K} \\
 &e = 0.8 \quad \sigma = 6 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \\
 &\frac{Q}{t} = Ae \sigma T^4 = 12 \times 10^{-4} \times 0.8 \times 6 \times 10^{-8} \times (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42
 \end{aligned}$$

41. E → Energy radiated per unit area per unit time

Rate of heat flow → Energy radiated
(a) Per time = E × A

$$\text{So, } E_{AI} = \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4} \quad \therefore 1:4$$

(b) Emissivity of both are same

$$\begin{aligned}
 &= \frac{m_1 S_1 dT_1}{m_2 S_2 dT_2} = 1 \\
 \Rightarrow \frac{dT_1}{dT_2} &= \frac{m_2 S_2}{m_1 S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1 : 2 : 9
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \frac{Q}{t} &= Ae \sigma T^4 \\
 \Rightarrow T^4 &= \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}} \\
 \Rightarrow T &= 1697.0 \approx 1700 \text{ K} \\
 43. \quad (a) A &= 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2, \quad T = 57^\circ\text{C} = 330 \text{ K} \\
 E &= A \sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42 \text{ J} \\
 (b) \frac{E}{t} &= A\sigma e(T_1^4 - T_2^4), \quad A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2 \\
 \sigma &= 6 \times 10^{-8} \quad T_1 = 473 \text{ K}, \quad T_2 = 330 \text{ K} \\
 &= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4] \\
 &= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}] \\
 &= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58 \text{ W} \quad \text{from the ball.}
 \end{aligned}$$

49. $\sigma = 6 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

$L = 20 \text{ cm} = 0.2 \text{ m}$, $K = ?$

$$\Rightarrow E = \frac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$$

$$\Rightarrow K = \frac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \frac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$$

$$\Rightarrow K = 73.993 \approx 74.$$

50. $v = 100 \text{ cc}$

$\Delta\theta = 5^\circ\text{C}$

$t = 5 \text{ min}$

For water

$$\frac{mS\Delta\theta}{dt} = \frac{KA}{l} \Delta\theta$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 1000 \times 4200}{5} = \frac{KA}{l}$$

For Kerosene

$$\frac{ms}{at} = \frac{KA}{l}$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{KA}{l}$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{100 \times 10^{-3} \times 1000 \times 4200}{5}$$

$$\Rightarrow T = \frac{5 \times 800 \times 2100}{1000 \times 4200} = 2 \text{ min}$$

51. 50°C 45°C 40°C

Let the surrounding temperature be $T^\circ\text{C}$

$$\text{Avg. } t = \frac{50 + 45}{2} = 47.5$$

Avg. temp. diff. from surrounding

$$T = 47.5 - T$$

$$\text{Rate of fall of temp} = \frac{50 - 45}{5} = 1^\circ\text{C/mm}$$

From Newton's Law

$$1^\circ\text{C/mm} = bA \times t$$

$$\Rightarrow bA = \frac{1}{t} = \frac{1}{47.5 - T} \quad \dots(1)$$

In second case,

$$\text{Avg. temp} = \frac{40 + 45}{2} = 42.5$$

Avg. temp. diff. from surrounding

$$t' = 42.5 - T$$

$$\text{Rate of fall of temp} = \frac{45 - 40}{8} = \frac{5}{8}^\circ\text{C/mm}$$

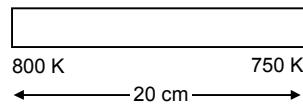
From Newton's Law

$$\frac{5}{B} = bAt'$$

$$\Rightarrow \frac{5}{8} = \frac{1}{(47.5 - T)} \times (42.5 - T)$$

By C & D [Componendo & Dividendo method]

We find, $T = 34.1^\circ\text{C}$



300 K

55. $\frac{d\theta}{dt} = -K(T - T_0)$

Temp. at $t = 0$ is θ_1

(a) Max. Heat that the body can loose $= \Delta Q_m = ms(\theta_1 - \theta_0)$

(\therefore as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0) \times 9}{10}$$

If it takes time t_1 , for this process, the temp. at t_1

$$= \theta_1 - (\theta_1 - \theta_0) \frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10} \times 1$$

Now, $\frac{d\theta}{dt} = -K(\theta - \theta_1)$

Let $\theta = \theta_1$ at $t = 0$; & θ be temp. at time t

$$\int_{\theta}^{\theta_1} \frac{d\theta}{\theta - \theta_0} = -K \int_0^t dt$$

or, $\ln \frac{\theta - \theta_0}{\theta_1 - \theta_0} = -Kt$

or, $\theta - \theta_0 = (\theta_1 - \theta_0) e^{-kt}$... (2)

Putting value in the Eq (1) and Eq (2)

$$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0) e^{-kt}$$

$$\Rightarrow t_1 = \frac{\ln 10}{k}$$

