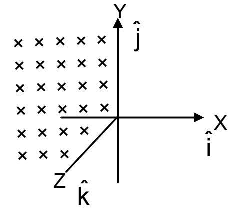
CHAPTER – 30
GAUSS'S LAW

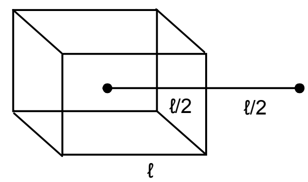


1. Given : $\vec{E} = 3/5\ E_0\ \hat{i} + 4/5\ E_0\ \hat{j}$

   $E_0 = 2.0 \times 10^3$ N/C The plane is parallel to yz-plane.

   Hence only 3/5 $E_0\ \hat{i}$ passes perpendicular to the plane whereas 4/5 $E_0\ \hat{j}$ goes parallel. Area = $0.2m^2$ (given)

   $\therefore$ Flux = $\vec{E} + \vec{A}$ = $3/5 \times 2 \times 10^3 \times 0.2 = 2.4 \times 10^2\ Nm^2/c = 240\ Nm^2/c$



2. Given length of rod = edge of cube = ℓ

   Portion of rod inside the cube = ℓ/2

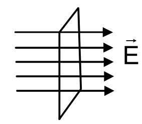
   Total charge = Q.

   Linear charge density = λ = Q/ℓ of rod.

   We know: Flux α charge enclosed.

   Charge enclosed in the rod inside the cube.

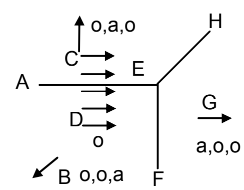
   = ℓ/2 ε_0 × Q/ℓ = Q/2 ε_0



3. As the electric field is uniform.

   Considering a perpendicular plane to it, we find that it is an equipotential surface. Hence there is no net current flow on that surface. Thus, net charge in that region is zero.

4. Given: E = $\frac{E_0\chi}{\ell}\hat{i}$ ℓ= 2 cm, a = 1cm.



   $E_0 = 5 \times 10^3$ N/C. From fig. We see that flux passes mainly through surface areas. ABDC & EFGH. As the AEFB & CHGD are paralled to the Flux. Again in ABDC a = 0; hence the Flux only passes through the surface are EFGH.

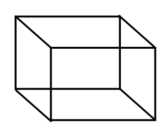
   E = $\frac{E_c x}{\ell}\hat{i}$

   Flux = $\frac{E_0\chi}{L}$ × Area = $\frac{5\times10^3\times a}{\ell}\times a^2 = \frac{5\times10^3\times a^3}{\ell} = \frac{5\times10^3\times(0.01)^{-3}}{2\times10^{-2}} = 2.5\times10^{-1}$

   Flux = $\frac{q}{\varepsilon_0}$ so, q = ε_0 × Flux

   = $8.85\times10^{-12}\times2.5\times10^{-1} = 2.2125\times10^{-12}$ c

5. According to Gauss's Law Flux = $\frac{q}{\varepsilon_0}$



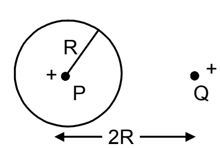
   Since the charge is placed at the centre of the cube. Hence the flux passing through the six surfaces = $\frac{Q}{6\varepsilon_0}\times 6 = \frac{Q}{\varepsilon_0}$

6. Given – A charge is placed o a plain surface with area = a^2, about a/2 from its centre.

   Assumption : let us assume that the given plain forms a surface of an imaginary cube. Then the charge is found to be at the centre of the cube.

   Hence flux through the surface = $\frac{Q}{\varepsilon_0}\times\frac{1}{6} = \frac{Q}{6\varepsilon_0}$

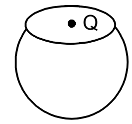
7. Given: Magnitude of the two charges placed = 10^{-7}c.



   We know: from Gauss's law that the flux experienced by the sphere is only due to the internal charge and not by the external one.

   Now $\oint\vec{E}.\vec{ds} = \frac{Q}{\varepsilon_0} = \frac{10^{-7}}{8.85\times10^{-12}}$ = 1.1×10^4 N-m^2/C.

8. We know: For a spherical surface

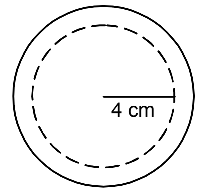


Flux = $\oint \vec{E}.ds = \frac{q}{\varepsilon_0}$ [by Gauss law]

Hence for a hemisphere = total surface area = $\frac{q}{\varepsilon_0} \times \frac{1}{2} = \frac{q}{2\varepsilon_0}$

9. Given: Volume charge density = 2.0×10^{-4} c/m^3

In order to find the electric field at a point 4cm = 4×10^{-2} m from the centre let us assume a concentric spherical surface inside the sphere.



Now, $\oint E.ds = \frac{q}{\varepsilon_0}$

But $\sigma = \frac{q}{4/3\pi R^3}$ so, q = $\sigma \times 4/3\ \pi\ R^3$

Hence = $\frac{\sigma \times 4/3 \times 22/7 \times (4 \times 10^{-2})^3}{\varepsilon_0} \times \frac{1}{4 \times 22/7 \times (4 \times 10^{-2})^2}$

= 2.0×10^{-4} 1/3 × 4 × 10^{-2} × $\frac{1}{8.85 \times 10^{-12}}$ = 3.0×10^5 N/C

10. Charge present in a gold nucleus = $79 \times 1.6 \times 10^{-19}$ C

Since the surface encloses all the charges we have:

(a) $\oint \vec{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0} = \frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}}$

E = $\frac{q}{\varepsilon_0 ds} = \frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}} \times \frac{1}{4 \times 3.14 \times (7 \times 10^{-15})^2}$ [$\because$ area = $4\pi r^2$]

= 2.3195131×10^{21} N/C

(b) For the middle part of the radius. Now here r = $7/2 \times 10^{-15}$m

Volume = 4/3 π r^3 = $\frac{48}{3} \times \frac{22}{7} \times \frac{343}{8} \times 10^{-45}$

Charge enclosed = ζ × volume [ζ : volume charge density]

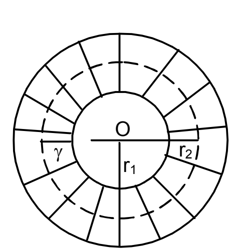
But $\zeta = \frac{\text{Net charge}}{\text{Net volume}} = \frac{7.9 \times 1.6 \times 10^{-19}\,c}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}}$

Net charged enclosed = $\frac{7.9 \times 1.6 \times 10^{-19}}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}} \times \frac{4}{3}\pi \times \frac{343}{8} \times 10^{-45} = \frac{7.9 \times 1.6 \times 10^{-19}}{8}$

$\oint \vec{E}\overrightarrow{ds} = \frac{q\ \text{enclosed}}{\varepsilon_0}$

$\Rightarrow$ E = $\frac{7.9 \times 1.6 \times 10^{-19}}{8 \times \varepsilon_0 \times S} = \frac{7.9 \times 1.6 \times 10^{-19}}{8 \times 8.85 \times 10^{-12} \times 4\pi \times \frac{49}{4} \times 10^{-30}}$ = 1.159×10^{21}N/C

11. Now, Volume charge density = $\frac{Q}{\frac{4}{3} \times \pi \times \left(r_2^{\ 3} - r_1^{\ 3}\right)}$



$\therefore$ $\zeta = \frac{3Q}{4\pi\left(r_2^{\ 3} - r_1^{\ 3}\right)}$

Again volume of sphere having radius x = $\frac{4}{3}\pi x^3$

Now charge enclosed by the sphere having radius

$$\chi = \left(\frac{4}{3}\pi\chi^3 - \frac{4}{3}\pi r_1^3\right) \times \frac{Q}{\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3} = Q\left(\frac{\chi^3 - r_1^3}{r_2^3 - r_1^3}\right)$$

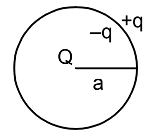
Applying Gauss's law – $E \times 4\pi\chi^2 = \dfrac{q\ \text{enclosed}}{\varepsilon_0}$

$$\Rightarrow E = \frac{Q}{\varepsilon_0}\left(\frac{\chi^3 - r_1^3}{r_2^3 - r_1^3}\right) \times \frac{1}{4\pi\chi^2} = \frac{Q}{4\pi\varepsilon_0\chi^2}\left(\frac{\chi^3 - r_1^3}{r_2^3 - r_1^3}\right)$$

12. Given: The sphere is uncharged metallic sphere.

Due to induction the charge induced at the inner surface = –Q, and that outer surface = +Q.

(a) Hence the surface charge density at inner and outer surfaces = $\dfrac{\text{charge}}{\text{total surface area}}$



$= -\dfrac{Q}{4\pi a^2}$ and $\dfrac{Q}{4\pi a^2}$ respectively.

(b) Again if another charge 'q' is added to the surface. We have inner surface charge density = $-\dfrac{Q}{4\pi a^2}$,

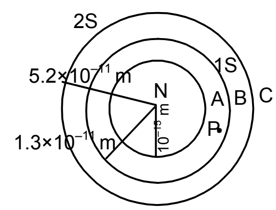
because the added charge does not affect it.

On the other hand the external surface charge density = $Q + \dfrac{q}{4\pi a^2}$ as the 'q' gets added up.

(c) For electric field let us assume an imaginary surface area inside the sphere at a distance 'x' from centre. This is same in both the cases as the 'q' in ineffective.

Now, $\oint E.ds = \dfrac{Q}{\varepsilon_0}$ So, $E = \dfrac{Q}{\varepsilon_0} \times \dfrac{1}{4\pi x^2} = \dfrac{Q}{4\pi\varepsilon_0 x^2}$

13. (a) Let the three orbits be considered as three concentric spheres A, B & C.



Now, Charge of 'A' = $4 \times 1.6 \times 10^{-16}$ c

Charge of 'B' = $2 \times 1.6 \times 10^{-16}$ c

Charge of 'C' = $2 \times 1.6 \times 10^{-16}$ c

As the point 'P' is just inside 1s, so its distance from centre = 1.3×10^{-11} m

Electric field = $\dfrac{Q}{4\pi\varepsilon_0 x^2} = \dfrac{4 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (1.3 \times 10^{-11})^2} = 3.4 \times 10^{13}$ N/C

(b) For a point just inside the 2 s cloud

Total charge enclosed = $4 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 2 \times 1.6 \times 10^{-19}$

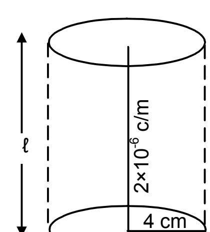
Hence, Electric filed,

$$\vec{E} = \frac{2 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (5.2 \times 10^{-11})^2} = 1.065 \times 10^{12} \text{ N/C} \approx 1.1 \times 10^{12} \text{ N/C}$$

14. Drawing an electric field around the line charge we find a cylinder of radius 4×10^{-2} m.

Given: λ = linear charge density

Let the length be $\ell = 2 \times 10^{-6}$ c/m



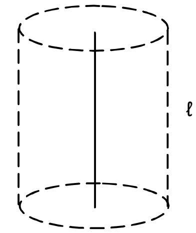
We know $\oint E.dl = \dfrac{Q}{\varepsilon_0} = \dfrac{\lambda\ell}{\varepsilon_0}$

$\Rightarrow E \times 2\pi r \ell = \dfrac{\lambda\ell}{\varepsilon_0} \Rightarrow E = \dfrac{\lambda}{\varepsilon_0 \times 2\pi r}$

For, $r = 2 \times 10^{-2}$ m & $\lambda = 2 \times 10^{-6}$ c/m

$$\Rightarrow E = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12} \times 2 \times 3.14 \times 2 \times 10^{-2}} = 8.99 \times 10^5 \text{ N/C} \approx 9 \times 10^5 \text{ N/C}$$

15. Given :
$\lambda = 2 \times 10^{-6}$ c/m
For the previous problem.
$E = \frac{\lambda}{\epsilon_0 \, 2\pi r}$ for a cylindrical electricfield.
Now, For experienced by the electron due to the electric filed in wire = centripetal force.

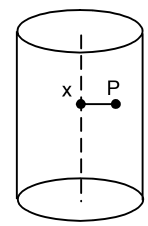


Eq = mv^2 $\left[\begin{array}{l} \text{we know}, m_e = 9.1\times10^{-31}\text{kg}, \\ v_e = ?,\ r = \text{assumed radius} \end{array}\right]$

$\Rightarrow \frac{1}{2}$ Eq = $\frac{1}{2}\frac{mv^2}{r}$

$\Rightarrow$ KE = 1/2 × E × q × r = $\frac{1}{2} \times \frac{\lambda}{\varepsilon_0 2\pi r} \times 1.6 \times 10^{-19} = 2.88 \times 10^{-17}$ J.

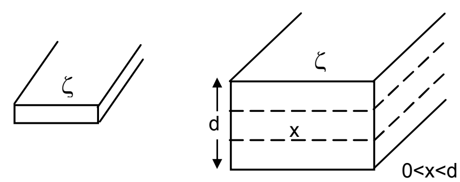
16. Given: Volume charge density = ζ
Let the height of cylinder be h.
$\therefore$ Charge Q at P = $\zeta \times 4\pi\chi^2 \times h$



For electric field $\oint E.ds = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{\varepsilon_0 \times ds} = \frac{\zeta \times 4\pi\chi^2 \times h}{\varepsilon_0 \times 2 \times \pi \times \chi \times h} = \frac{2\zeta\chi}{\varepsilon_0}$

17. $\oint E.dA = \frac{Q}{\varepsilon_0}$



Let the area be A.
Uniform change distribution density is ζ
Q = ζA

$E = \frac{Q}{\varepsilon_0} \times dA = \frac{\zeta \times a \times \chi}{\varepsilon_0 \times A} = \frac{\zeta\chi}{\varepsilon_0}$

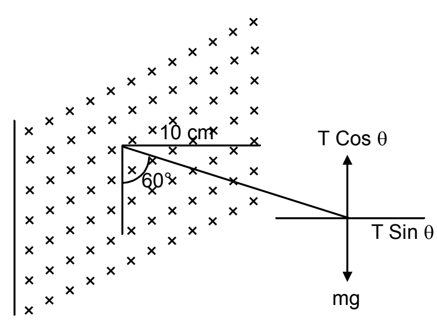
18. $Q = -2.0 \times 10^{-6}$C Surface charge density = 4×10^{-6} C/m^2

We know $\vec{E}$ due to a charge conducting sheet = $\frac{\sigma}{2\varepsilon_0}$

Again Force of attraction between particle & plate

= Eq = $\frac{\sigma}{2\varepsilon_0} \times q = \frac{4\times10^{-6}\times2\times10^{-6}}{2\times8\times10^{-12}}$ = 0.452N

19. Ball mass = 10g
Charge = 4×10^{-6} c
Thread length = 10 cm
Now from the fig, T cosθ = mg
T sinθ = electric force



Electric force = $\frac{\sigma q}{2\varepsilon_0}$ (σ surface charge density)

T sinθ = $\frac{\sigma q}{2\varepsilon_0}$, T cos$\theta$ = mg

Tan $\theta = \frac{\sigma q}{2mg\varepsilon_0}$

$\sigma = \frac{2mg\varepsilon_0 \tan\theta}{q} = \frac{2\times8.85\times10^{-12}\times10\times10^{-3}\times9.8\times1.732}{4\times10^{-6}} = 7.5 \times 10^{-7}$ C/m^2

20. (a) Tension in the string in Equilibrium

$T \cos 60° = mg$

$$\Rightarrow T = \frac{mg}{\cos 60°} = \frac{10 \times 10^{-3} \times 10}{1/2} = 10^{-1} \times 2 = 0.20 \text{ N}$$

(b) Straingtening the same figure.

Now the resultant for 'R'

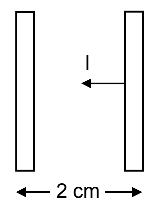
Induces the acceleration in the pendulum.

$$T = 2 \times \pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{\left[g^2 + \left(\frac{\sigma q}{2\varepsilon_0 m}\right)^2\right]^{1/2}}} = 2\pi\sqrt{\frac{\ell}{\left[100 + \left(0.2 \times \frac{\sqrt{3}}{2 \times 10^{-2}}\right)^2\right]^{1/2}}}$$

$$= 2\pi\sqrt{\frac{\ell}{(100+300)^{1/2}}} = 2\pi\sqrt{\frac{\ell}{20}} = 2 \times 3.1416 \times \sqrt{\frac{10 \times 10^{-2}}{20}} = 0.45 \text{ sec.}$$

21. $s = 2\text{cm} = 2 \times 10^{-2}\text{m}$, $u = 0$, $a = ?$ $t = 2\mu s = 2 \times 10^{-6}\text{s}$

Acceleration of the electron, $s = (1/2)\, at^2$



$$2 \times 10^{-2} = (1/2) \times a \times (2 \times 10^{-6})^2 \Rightarrow a = \frac{2 \times 2 \times 10^{-2}}{4 \times 10^{-12}} \Rightarrow a = 10^{10} \text{ m/s}^2$$

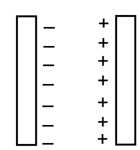
The electric field due to charge plate $= \frac{\sigma}{\varepsilon_0}$

Now, electric force $= \frac{\sigma}{\varepsilon_0} \times q$ = acceleration $= \frac{\sigma}{\varepsilon_0} \times \frac{q}{m_e}$

Now $\frac{\sigma}{\varepsilon_0} \times \frac{q}{m_e} = 10^{10}$

$$\Rightarrow \sigma = \frac{10^{10} \times \varepsilon_0 \times m_e}{q} = \frac{10^{10} \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$$

$= 50.334 \times 10^{-14} = 0.50334 \times 10^{-12}\ \text{c/m}^2$

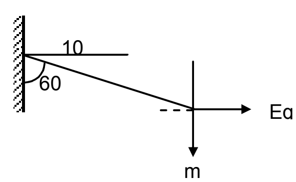


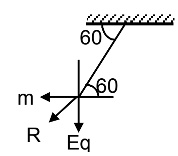
22. Given: Surface density $= \sigma$

(a) & (c) For any point to the left & right of the dual plater, the electric field is zero.

As there are no electric flux outside the system.

(b) For a test charge put in the middle.



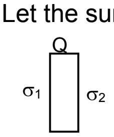


It experiences a fore $\frac{\sigma q}{2\varepsilon_0}$ towards the (-ve) plate.

Hence net electric field $\frac{1}{q}\left(\frac{\sigma q}{2\varepsilon_0} + \frac{\sigma q}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$

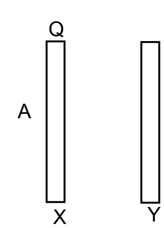
23. (a) For the surface charge density of a single plate.

Let the surface charge density at both sides be σ_1 & σ_2



= Now, electric field at both ends.

$= \frac{\sigma_1}{2\varepsilon_0}$ & $\frac{\sigma_2}{2\varepsilon_0}$



Due to a net balanced electric field on the plate $\frac{\sigma_1}{2\varepsilon_0}$ & $\frac{\sigma_2}{2\varepsilon_0}$

$\therefore \sigma_1 = \sigma_2$ So, $q_1 = q_2 = Q/2$

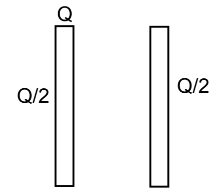
$\therefore$ Net surface charge density = Q/2A

(b) Electric field to the left of the plates = $\frac{\sigma}{\varepsilon_0}$

Since σ = Q/2A   Hence Electricfield = $Q/2A\varepsilon_0$

This must be directed toward left as 'X' is the charged plate.

(c) & (d) Here in both the cases the charged plate 'X' acts as the only source of electric field, with (+ve) in the inner side and 'Y' attracts towards it with (-ve) he in its inner side. So for the middle portion E = $\frac{Q}{2A\varepsilon_0}$ towards right.



(d) Similarly for extreme right the outerside of the 'Y' plate acts as positive and hence it repels to the right with E = $\frac{Q}{2A\varepsilon_0}$

24. Consider the Gaussian surface the induced charge be as shown in figure.

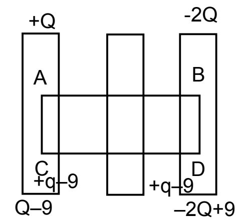
The net field at P due to all the charges is Zero.

$\therefore$ –2Q +9/2A ε_0 (left) +9/2A ε_0 (left) + 9/2A ε_0 (right) + Q – 9/2A ε_0 (right) = 0

$\Rightarrow$ –2Q + 9 – Q + 9 = 0 $\Rightarrow$ 9 = 3/2 Q

$\therefore$ charge on the right side of right most plate

= –2Q + 9 = – 2Q + 3/2 Q = – Q/2



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