

ELECTROMAGNETIC WAVES CHAPTER - 40

$$1. \quad \frac{\epsilon_0 d\phi_E}{dt} = \frac{\epsilon_0 EA}{4\pi \epsilon_0 r^2}$$

$$= \frac{M^{-1}L^{-3}T^4A^2}{M^{-1}L^{-3}A^2} \times \frac{A^1T^1}{L^2} \times \frac{L^2}{T} = A^1$$

= (Current) (proved).

$$2. \quad E = \frac{Kq}{x^2}, \text{ [from coulomb's law]}$$

$$\phi_E = EA = \frac{KqA}{x^2}$$

$$i_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{KqA}{x^2} \right) = \epsilon_0 KqA \frac{d}{dt} x^{-2}$$

$$= \epsilon_0 \times \frac{1}{4\pi \epsilon_0} \times q \times A \times -2 \times x^{-3} \times \frac{dx}{dt} = \frac{qAv}{2\pi x^3}$$

$$3. \quad E = \frac{Q}{\epsilon_0 A} \text{ (Electric field)}$$

$$\phi = E.A. = \frac{Q}{\epsilon_0 A} \frac{A}{2} = \frac{Q}{\epsilon_0 2}$$

$$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0 2} \right) = \frac{1}{2} \left(\frac{dQ}{dt} \right)$$

$$= \frac{1}{2} \frac{d}{dt} (ECe^{-t/RC}) = \frac{1}{2} EC - \frac{1}{RC} e^{-t/RC} = \frac{E}{2R} e^{-t/RC}$$

$$4. \quad E = \frac{Q}{\epsilon_0 A} \text{ (Electric field)}$$

$$\phi = E.A. = \frac{Q}{\epsilon_0 A} \frac{A}{2} = \frac{Q}{\epsilon_0 2}$$

$$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0 2} \right) = \frac{1}{2} \left(\frac{dQ}{dt} \right)$$

$$5. \quad B = \mu_0 H$$

$$\Rightarrow H = \frac{B}{\mu_0}$$

$$\frac{E_0}{H_0} = \frac{B_0 / (\mu_0 \epsilon_0 C)}{B_0 / \mu_0} = \frac{1}{\epsilon_0 C}$$

$$= \frac{1}{8.85 \times 10^{-12} \times 3 \times 10^8} = 376.6 \Omega = 377 \Omega.$$

$$\text{Dimension } \frac{1}{\epsilon_0 C} = \frac{1}{[LT^{-1}][M^{-1}L^{-3}T^4A^2]} = \frac{1}{M^{-1}L^{-2}T^3A^2} = M^1L^2T^{-3}A^{-2} = [R].$$

$$6. \quad E_0 = 810 \text{ V/m}, B_0 = ?$$

We know, $B_0 = \mu_0 \epsilon_0 C E_0$

Putting the values,

$$B_0 = 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 3 \times 10^8 \times 810$$

$$= 27010.9 \times 10^{-10} = 2.7 \times 10^{-6} \text{ T} = 2.7 \mu\text{T}.$$

7. $B = (200 \mu\text{T}) \sin [(4 \times 10^{15} \text{ s}^{-1}) (t - x/c)]$

a) $B_0 = 200 \mu\text{T}$

$$E_0 = c \times B_0 = 200 \times 10^{-6} \times 3 \times 10^8 = 6 \times 10^4$$

b) Average energy density = $\frac{1}{2\mu_0} B_0^2 = \frac{(200 \times 10^{-6})^2}{2 \times 4\pi \times 10^{-7}} = \frac{4 \times 10^{-8}}{8\pi \times 10^{-7}} = \frac{1}{20\pi} = 0.0159 = 0.016.$

8. $I = 2.5 \times 10^{14} \text{ W/m}^2$

We know, $I = \frac{1}{2} \epsilon_0 E_0^2 c$

$$\Rightarrow E_0^2 = \frac{2I}{\epsilon_0 c} \quad \text{or } E_0 = \sqrt{\frac{2I}{\epsilon_0 c}}$$

$$E_0 = \sqrt{\frac{2 \times 2.5 \times 10^{14}}{8.85 \times 10^{-12} \times 3 \times 10^8}} = 0.4339 \times 10^9 = 4.33 \times 10^8 \text{ N/c.}$$

$$B_0 = \mu_0 \epsilon_0 c E_0$$

$$= 4 \times 3.14 \times 10^{-7} \times 8.854 \times 10^{-12} \times 3 \times 10^8 \times 4.33 \times 10^8 = 1.44 \text{ T.}$$

9. Intensity of wave = $\frac{1}{2} \epsilon_0 E_0^2 c$

$$\epsilon_0 = 8.85 \times 10^{-12}; E_0 = ?; c = 3 \times 10^8, I = 1380 \text{ W/m}^2$$

$$1380 = \frac{1}{2} \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$$

$$\Rightarrow E_0^2 = \frac{2 \times 1380}{8.85 \times 3 \times 10^{-4}} = 103.95 \times 10^4$$

$$\Rightarrow E_0 = 10.195 \times 10^2 = 1.02 \times 10^3$$

$$E_0 = B_0 c$$

$$\Rightarrow B_0 = E_0/c = \frac{1.02 \times 10^3}{3 \times 10^8} = 3.398 \times 10^{-5} = 3.4 \times 10^{-5} \text{ T.}$$

