

X - RAYS CHAPTER 44

1. $\lambda = 0.1 \text{ nm}$

a) Energy = $\frac{hc}{\lambda} = \frac{1242 \text{ eV}\cdot\text{nm}}{0.1 \text{ nm}}$

= 12420 eV = 12.42 KeV = 12.4 keV.

b) Frequency = $\frac{C}{\lambda} = \frac{3 \times 10^8}{0.1 \times 10^{-9}} = \frac{3 \times 10^8}{10^{-10}} = 3 \times 10^{18} \text{ Hz}$

c) Momentum = $E/C = \frac{12.4 \times 10^3 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 6.613 \times 10^{-24} \text{ kg}\cdot\text{m/s} = 6.62 \times 10^{-24} \text{ kg}\cdot\text{m/s}$.

2. Distance = 3 km = $3 \times 10^3 \text{ m}$

$C = 3 \times 10^8 \text{ m/s}$

$t = \frac{\text{Dist}}{\text{Speed}} = \frac{3 \times 10^3}{3 \times 10^8} = 10^{-5} \text{ sec.}$

$\Rightarrow 10 \times 10^{-8} \text{ sec} = 10 \mu\text{s}$ in both case.

3. $V = 30 \text{ KV}$

$\lambda = \frac{hc}{E} = \frac{hc}{eV} = \frac{1242 \text{ eV}\cdot\text{nm}}{e \times 30 \times 10^3} = 41.4 \times 10^{-4} \text{ nm} = 41.4 \text{ Pm.}$

4. $\lambda = 0.10 \text{ nm} = 10^{-10} \text{ m}; \quad h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

$C = 3 \times 10^8 \text{ m/s}; \quad e = 1.6 \times 10^{-19} \text{ C}$

$\lambda_{\min} = \frac{hc}{eV} \quad \text{or} \quad V = \frac{hc}{e\lambda}$

= $\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10^{-10}} = 12.43 \times 10^3 \text{ V} = 12.4 \text{ KV.}$

Max. Energy = $\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} = 19.89 \times 10^{-18} = 1.989 \times 10^{-15} = 2 \times 10^{-15} \text{ J.}$

5. $\lambda = 80 \text{ pm}, E = \frac{hc}{\lambda} = \frac{1242}{80 \times 10^{-3}} = 15.525 \times 10^3 \text{ eV} = 15.5 \text{ KeV}$

6. We know $\lambda = \frac{hc}{V}$

Now $\lambda = \frac{hc}{1.01V} = \frac{\lambda}{1.01}$

$\lambda - \lambda' = \frac{0.01}{1.01} \lambda.$

% change of wave length = $\frac{0.01 \times \lambda}{1.01 \times \lambda} \times 100 = \frac{1}{1.01} = 0.9900 = 1\%$.

7. $d = 1.5 \text{ m}, \lambda = 30 \text{ pm} = 30 \times 10^{-3} \text{ nm}$

$E = \frac{hc}{\lambda} = \frac{1242}{30 \times 10^{-3}} = 41.4 \times 10^3 \text{ eV}$

Electric field = $\frac{V}{d} = \frac{41.4 \times 10^3}{1.5} = 27.6 \times 10^3 \text{ V/m} = 27.6 \text{ KV/m.}$

8. Given $\lambda' = \lambda - 26 \text{ pm}, V' = 1.5 \text{ V}$

Now, $\lambda = \frac{hc}{eV}, \quad \lambda' = \frac{hc}{eV'}$

or $\lambda V = \lambda' V'$

$\Rightarrow \lambda V = (\lambda - 26 \times 10^{-12}) \times 1.5 \text{ V}$

$$\Rightarrow \lambda = 1.5 \lambda - 1.5 \times 26 \times 10^{-12}$$

$$\Rightarrow \lambda = \frac{39 \times 10^{-12}}{0.5} = 78 \times 10^{-12} \text{ m}$$

$$V = \frac{hc}{e\lambda} = \frac{6.63 \times 3 \times 10^{-34} \times 10^8}{1.6 \times 10^{-19} \times 78 \times 10^{-12}} = 0.15937 \times 10^5 = 15.93 \times 10^3 \text{ V} = 15.93 \text{ KV.}$$

9. $V = 32 \text{ KV} = 32 \times 10^3 \text{ V}$

When accelerated through 32 KV

$$E = 32 \times 10^3 \text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{1242}{32 \times 10^3} = 38.8 \times 10^{-3} \text{ nm} = 38.8 \text{ pm.}$$

10. $\lambda = \frac{hc}{eV}$; $V = 40 \text{ kV}$, $f = 9.7 \times 10^{18} \text{ Hz}$

$$\text{or, } \frac{h}{c} = \frac{h}{eV} ; \text{ or, } \frac{i}{f} = \frac{h}{eV} ; \text{ or } h = \frac{eV}{f} \text{ V-s}$$

$$= \frac{eV}{f} \text{ V-s} = \frac{40 \times 10^3}{9.7 \times 10^{18}} = 4.12 \times 10^{-15} \text{ eV-s.}$$

11. $V = 40 \text{ KV} = 40 \times 10^3 \text{ V}$

$$\text{Energy} = 40 \times 10^3 \text{ eV}$$

$$\text{Energy utilized} = \frac{70}{100} \times 40 \times 10^3 = 28 \times 10^3 \text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{1242 \text{ - eV nm}}{28 \times 10^3 \text{ eV}} \Rightarrow 44.35 \times 10^{-3} \text{ nm} = 44.35 \text{ pm.}$$

For other wavelengths,

$$E = 70\% \text{ (left over energy)} = \frac{70}{100} \times (40 - 28) \times 10^3 = 84 \times 10^2.$$

$$\lambda' = \frac{hc}{E} = \frac{1242}{8.4 \times 10^3} = 147.86 \times 10^{-3} \text{ nm} = 147.86 \text{ pm} = 148 \text{ pm.}$$

For third wavelength,

$$E = \frac{70}{100} = (12 - 8.4) \times 10^3 = 7 \times 3.6 \times 10^2 = 25.2 \times 10^2$$

$$\lambda' = \frac{hc}{E} = \frac{1242}{25.2 \times 10^2} = 49.2857 \times 10^{-2} \text{ nm} = 493 \text{ pm.}$$

12. $K_\alpha = 21.3 \times 10^{-12} \text{ pm}$, Now, $E_K - E_L = \frac{1242}{21.3 \times 10^{-3}} = 58.309 \text{ kev}$

$$E_L = 11.3 \text{ kev,}$$

$$E_K = 58.309 + 11.3 = 69.609 \text{ kev}$$

$$\text{Now, } V_e = 69.609 \text{ KeV, or } V = 69.609 \text{ KV.}$$

13. $\lambda = 0.36 \text{ nm}$

$$E = \frac{1242}{0.36} = 3450 \text{ eV } (E_M - E_K)$$

Energy needed to ionize an organ atom = 16 eV

Energy needed to knock out an electron from K-shell

$$= (3450 + 16) \text{ eV} = 3466 \text{ eV} = 3.466 \text{ KeV.}$$

14. $\lambda_1 = 887 \text{ pm}$

$$v = \frac{C}{\lambda} = \frac{3 \times 10^8}{887 \times 10^{-12}} = 3.382 \times 10^7 = 33.82 \times 10^{16} = 5.815 \times 10^8$$

$$\lambda_2 = 146 \text{ pm}$$

$$v = \frac{3 \times 10^8}{146 \times 10^{-12}} = 0.02054 \times 10^{20} = 2.054 \times 10^{18} = 1.4331 \times 10^9.$$

We know, $\sqrt{v} = a(z - b)$

$$\Rightarrow \frac{\sqrt{5.815 \times 10^8}}{\sqrt{1.4331 \times 10^9}} = \frac{a(13 - b)}{a(30 - b)}$$

$$\Rightarrow \frac{13 - b}{30 - b} = \frac{5.815 \times 10^{-1}}{1.4331} = 0.4057.$$

$$\Rightarrow 30 \times 0.4057 - 0.4057 b = 13 - b$$

$$\Rightarrow 12.171 - 0.4.57 b + b = 13$$

$$\Rightarrow b = \frac{0.829}{0.5943} = 1.39491$$

$$\Rightarrow a = \frac{5.815 \times 10^8}{11.33} = 0.51323 \times 10^8 = 5 \times 10^7.$$

For 'Fe',

$$\sqrt{v} = 5 \times 10^7 (26 - 1.39) = 5 \times 24.61 \times 10^7 = 123.05 \times 10^7$$

$$c/\lambda = 15141.3 \times 10^{14}$$

$$= \lambda = \frac{3 \times 10^8}{15141.3 \times 10^{14}} = 0.000198 \times 10^{-6} \text{ m} = 198 \times 10^{-12} = 198 \text{ pm}.$$

15. $E = 3.69 \text{ kev} = 3690 \text{ eV}$

$$\lambda = \frac{hc}{E} = \frac{1242}{3690} = 0.33658 \text{ nm}$$

$$\sqrt{c/\lambda} = a(z - b); \quad a = 5 \times 10^7 \sqrt{\text{Hz}}, \quad b = 1.37 \text{ (from previous problem)}$$

$$\sqrt{\frac{3 \times 10^8}{0.34 \times 10^{-9}}} = 5 \times 10^7 (Z - 1.37) \Rightarrow \sqrt{8.82 \times 10^{17}} = 5 \times 10^7 (Z - 1.37)$$

$$\Rightarrow 9.39 \times 10^8 = 5 \times 10^7 (Z - 1.37) \Rightarrow 93.9 / 5 = Z - 1.37$$

$$\Rightarrow Z = 20.15 = 20$$

\therefore The element is calcium.

16. K_B radiation is when the e jumps from $n = 3$ to $n = 1$ (here n is principal quantum no)

$$\Delta E = hv = Rhc (z - h)^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \sqrt{v} = \sqrt{\frac{9RC}{8}} (z - h)$$

$$\therefore \sqrt{v} \propto z$$

Second method :

We can directly get value of v by

$$hv = \text{Energy}$$

$$\Rightarrow v = \frac{\text{Energy(in kev)}}{h}$$

This we have to find out \sqrt{v} and draw the same graph as above.

17. $b = 1$

For $\propto a$ (57)

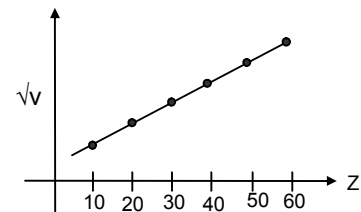
$$\sqrt{v} = a (Z - b)$$

$$\Rightarrow \sqrt{v} = a (57 - 1) = a \times 56 \quad \dots(1)$$

For Cu(29)

$$\sqrt{1.88 \times 10^{78}} = a(29 - 1) = 28 a \quad \dots(2)$$

dividing (1) and (2)



$$\sqrt{\frac{v}{1.88 \times 10^{18}}} = \frac{a \times 56}{a \times 28} = 2.$$

$$\Rightarrow v = 1.88 \times 10^{18} (2)^2 = 4 \times 1.88 \times 10^{18} = 7.52 \times 10^{18} \text{ Hz.}$$

18. $K_{\alpha} = E_K - E_L \quad \dots(1) \quad \lambda K_{\alpha} = 0.71 \text{ \AA}$

$K_{\beta} = E_K - E_M \quad \dots(2) \quad \lambda K_{\beta} = 0.63 \text{ \AA}$

$L_{\alpha} = E_L - E_M \quad \dots(3)$

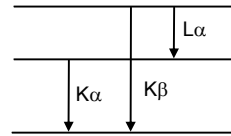
Subtracting (2) from (1)

$$K_{\alpha} - K_{\beta} = E_M - E_L = -L_{\alpha}$$

$$\text{or, } L_{\alpha} = K_{\beta} - K_{\alpha} = \frac{3 \times 10^8}{0.63 \times 10^{-10}} - \frac{3 \times 10^8}{0.71 \times 10^{-10}}$$

$$= 4.761 \times 10^{18} - 4.225 \times 10^{18} = 0.536 \times 10^{18} \text{ Hz.}$$

$$\text{Again } \lambda = \frac{3 \times 10^8}{0.536 \times 10^{18}} = 5.6 \times 10^{-10} = 5.6 \text{ \AA}.$$

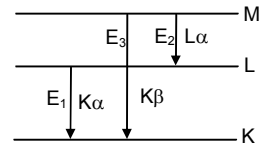


19. $E_1 = \frac{1242}{21.3 \times 10^{-3}} = 58.309 \times 10^3 \text{ eV}$

$$E_2 = \frac{1242}{141 \times 10^{-3}} = 8.8085 \times 10^3 \text{ eV}$$

$$E_3 = E_1 + E_2 \Rightarrow (58.309 + 8.809) \text{ eV} = 67.118 \times 10^3 \text{ eV}$$

$$\lambda = \frac{hc}{E_3} = \frac{1242}{67.118 \times 10^3} = 18.5 \times 10^{-3} \text{ nm} = 18.5 \text{ pm.}$$



20. $E_K = 25.31 \text{ KeV}, E_L = 3.56 \text{ KeV}, E_M = 0.530 \text{ KeV}$

$$K_{\alpha} = E_K - E_L = h\nu$$

$$\Rightarrow v = \frac{E_K - E_L}{h} = \frac{25.31 - 3.56}{4.14 \times 10^{-15}} \times 10^3 = 5.25 \times 10^{15} \text{ Hz}$$

$$K_{\beta} = E_K - E_M = h\nu$$

$$\Rightarrow v = \frac{E_K - E_M}{h} = \frac{25.31 - 0.53}{4.14 \times 10^{-15}} \times 10^3 = 5.985 \times 10^{18} \text{ Hz.}$$

21. Let for, k series emission the potential required = v

\therefore Energy of electrons = ev

This amount of energy ev = energy of L shell

The maximum potential difference that can be applied without emitting any electron is 11.3 ev.

22. $V = 40 \text{ KV}, i = 10 \text{ mA}$

1% of T_{KE} (Total Kinetic Energy) = X ray

$$i = ne \quad \text{or } n = \frac{10^{-2}}{1.6 \times 10^{-19}} = 0.625 \times 10^{17} \text{ no. of electrons.}$$

$$\text{KE of one electron} = eV = 1.6 \times 10^{-19} \times 40 \times 10^3 = 6.4 \times 10^{-15} \text{ J}$$

$$T_{KE} = 0.625 \times 6.4 \times 10^{17} \times 10^{-15} = 4 \times 10^2 \text{ J.}$$

a) Power emitted in X-ray = $4 \times 10^2 \times (-1/100) = 4 \text{ w}$

b) Heat produced in target per second = $400 - 4 = 396 \text{ J.}$

23. Heat produced/sec = 200 w

$$\Rightarrow \frac{neV}{t} = 200 \Rightarrow (ne/t)V = 200$$

$$\Rightarrow i = 200 / V = 10 \text{ mA.}$$

24. Given : $v = (25 \times 10^{14} \text{ Hz})(Z - 1)^2$

$$\text{Or } C/\lambda = 25 \times 10^{14} (Z - 1)^2$$

a) $\frac{3 \times 10^8}{78.9 \times 10^{-12} \times 25 \times 10^{14}} = (Z - 1)^2$

$$\text{or, } (Z - 1)^2 = 0.001520 \times 10^6 = 1520$$

$$\Rightarrow Z - 1 = 38.98 \text{ or } Z = 39.98 = 40. \text{ It is (Zr)}$$

$$b) \frac{3 \times 10^8}{146 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

$$\text{or, } (Z-1)^2 = 0.0008219 \times 10^6$$

$$\Rightarrow Z-1 = 28.669 \text{ or } Z = 29.669 = 30. \text{ It is (Zn).}$$

$$c) \frac{3 \times 10^8}{158 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

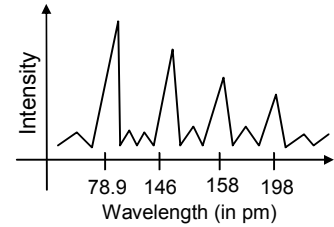
$$\text{or, } (Z-1)^2 = 0.0007594 \times 10^6$$

$$\Rightarrow Z-1 = 27.5589 \text{ or } Z = 28.5589 = 29. \text{ It is (Cu).}$$

$$d) \frac{3 \times 10^8}{198 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

$$\text{or, } (Z-1)^2 = 0.000606 \times 10^6$$

$$\Rightarrow Z-1 = 24.6182 \text{ or } Z = 25.6182 = 26. \text{ It is (Fe).}$$



25. Here energy of photon = E
 $E = 6.4 \text{ KeV} = 6.4 \times 10^3 \text{ eV}$

$$\text{Momentum of Photon} = E/C = \frac{6.4 \times 10^3}{3 \times 10^8} = 3.41 \times 10^{-24} \text{ m/sec.}$$

According to collision theory of momentum of photon = momentum of atom

$$\therefore \text{Momentum of Atom} = P = 3.41 \times 10^{-24} \text{ m/sec}$$

$$\therefore \text{Recoil K.E. of atom} = P^2 / 2m$$

$$\Rightarrow \frac{(3.41 \times 10^{-24})^2 \text{ eV}}{(2)(9.3 \times 10^{-26} \times 1.6 \times 10^{-19})} = 3.9 \text{ eV} [1 \text{ Joule} = 1.6 \times 10^{-19} \text{ eV}]$$

26. $V_0 \rightarrow$ Stopping Potential, $\lambda \rightarrow$ Wavelength, $eV_0 = h\nu - h\nu_0$

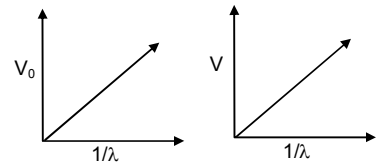
$$eV_0 = hc/\lambda \Rightarrow V_0\lambda = hc/e$$

$V \rightarrow$ Potential difference across X-ray tube, $\lambda \rightarrow$ Cut of wavelength

$$\lambda = hc / eV \quad \text{or} \quad V\lambda = hc / e$$

Slopes are same i.e. $V_0\lambda = V\lambda$

$$\frac{hc}{e} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} = 1.242 \times 10^{-6} \text{ Vm}$$



27. $\lambda = 10 \text{ pm} = 100 \times 10^{-12} \text{ m}$

$$D = 40 \text{ cm} = 40 \times 10^{-2} \text{ m}$$

$$\beta = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$$

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow d = \frac{\lambda D}{\beta} = \frac{100 \times 10^{-12} \times 40 \times 10^{-2}}{10^{-3} \times 0.1} = 4 \times 10^{-7} \text{ m.}$$

